KARIMPUR PANNADEVI COLLEGE
MATHEMATICS-HONOURS (Fourth Semester)
Internal Assessment, 2020

Duration: 2 **hrs.** **Full Marks: 30**

Group-A (CC-T-08)

Answer any two questions

Q.1) State and prove the Cauchy criterion for uniform convergence of series of functions. 5 marks

Q.2) Establish the Bessel's inequality related to Fourier series. 5 marks

Q.3) Define a piecewise continuous function. Prove or disprove that every bounded piecewise continuous function $f : [a, b] \to R$ is Riemann integrable. 5 marks

Q.4) State and prove Weierstrass M-test for series of functions. 5 marks

Group-B (CC-T-09)

Answer any two questions

Q.5) Let (a, b) be an interior point of domain of definition of a function f of two variables x, y. If $f_x(a, b)$ exists and $f_y(x, y)$ is continuous at (a, b), then prove that $f(x, y)$ is differentiable at (a, b). 5 marks

Q.6) If (a, b) be a point in the domain of definition of $f(x, y)$ such that $f_x(x, y)$ and $f_y(x, y)$ are differentiable at (a, b), prove that $f_{xy}(a, b) = f_{yx}(a, b)$. 5 marks

Q.7) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = L$ using the spherical polar coordinates. 5 marks

Q.8) Let $F = (P, Q)$ be a continuously differentiable function defined on a simply connected region D in $\mathbf{R}^2$. Show that $\int_C Pdx + Qdy = 0$ around every piecewise smooth closed curve C in D if and only if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, for all $(x, y) \in D$. 5 marks

Group-C (CC-T-10)

Answer any two questions

Q.9) Prove that every finite integral domain is a field. 5 marks

Q.10) If R is a commutative ring then the ideal P is a prime ideal if and only if the quotient ring R/P is an integral domain. 5 marks

Q.11) Prove that the characteristic of an integral domain is either zero or a prime number. 5 marks

Q.12) Prove that the ideal $p\mathbb{Z}$ in the ring $\mathbb{Z}$ is maximal if and only if p is a prime. 5 marks